03/24/2016

Empirical Validation of the Exponential Decay for Surplus CO_2

Ari Halperin
defyccc.com
Texas, USA
ah@defyccc.com

ABSTRACT

Surplus CO_2 is naturally removed from the atmosphere by natural sinks at a rate proportional to the surplus CO_2 concentration, on the multi-decadal scale. This result, analytically derived in (Halperin, *Simple Equation of Multi-Decadal Atmospheric Carbon Concentration Change*, 2015) is verified here by applying it to the pre-1958 data, which was not used in the original paper. The excellent match confirms the validity of the theoretical result. This paper also presents a more accurate estimate of the half-life of the surplus CO_2 concentration: 30-35 years. The correspondent equilibrium concentration is estimated to be in the range of 267-285 ppm (larger equilibrium concentrations correspond to lower half-lives). Also, the paper finds that if the natural sink rate did change in the past 150 years, it increased at least during the period prior to 1958. The paper uses CO_2 emissions data, corrected for some inaccuracies, introduced since 1992.

Cite as:

Halperin Ari, 2016. Empirical Validation of Exponential Decay for Surplus CO2, *defyccc.com*, defyccc.com/se2016

1. Introduction [1]

This paper follows up on (Halperin, *Simple Equation of Multi-Decadal Atmospheric Carbon Concentration Change*, 2015), referenced here as H2015, which contains a more detailed bibliography. The prior author's comment, (Halperin, *Comment #1 on Simple Equation of Multi-Decadal Atmospheric Carbon Concentration Change*, 2015) is referenced here as H2015#1. Texts of the Intergovernmental Panel on Climate Change (IPCC) are not relevant to the subject, in part because they subjugate physics and biology of the carbon cycle to the IPCC's political goals. See (Halperin, *The IPCC has been Deceiving the Public about the Carbon Cycle from the Start*, 2016) and the IPCC Disclaimer.

H2015 concludes that in response to a pulse release creating surplus atmospheric CO_2 concentration C(0), the surplus concentration would decrease over the following decades according to the formula of exponential decay:

$$\boldsymbol{C(t) = C(0)exp(-\lambda t)} \qquad \textbf{(1)}$$

where λ is a constant sink rate coefficient.

Considering the anthropogenic CO_2 release from fossil fuel combustion and cement manufacturing (at rate r_{ff}), land use change (at rate r_{luc}), and disregarding natural CO_2 fluctuations, gives the following formula for the evolution of the surplus CO_2 concentration in the atmosphere:

$$\boldsymbol{C'(t) = r_{ff}(t) + r_{luc}(t) - \lambda C(t)} \qquad \textbf{(2)}$$

with the initial condition of $C(0) \approx 0$ (near equilibrium) at some time before 1860. Discretization of formula (2) is used for the calculations in this paper. For readers' convenience, the half-life $h = ln(2)/\lambda$ is used throughout the paper.

2. Data

This paper uses all the same data sources as H2015 (Boden, Andres, & Marland, 2013), (Keeling, et al., 2001), (Keeling, Adams Jr., Ekdahl Jr., & Guenther, 1976), (Houghton, 2008), (Quéré, 2015), but also

[1] Supplementary material is available from http://defyccc.com/se2016.

includes the period from 1860 to 1958 for CO_2 emissions. Further, emissions data from 1992 is revised to what the author considers the most likely values. In particular:

- Land Use Change data from 1992 is completely disregarded because it was likely fabricated to a large extent, as shown in (Halperin, *Notes on FAO-FRA*, 2015). In a nutshell, it is hard to believe that Indonesian or Brazilian farmers or peasants ceased clearing forests for their needs just because 1992 Rio Summit or another meeting demanded it, even if they knew of these events. The emissions data was calculated based on a conservative assumption of moderate 1% growth.
- 2004-2011 fossil fuel emissions underreported by China were added, using (Boden & Marland, 2013) and the author's subjective judgement.
- Obviously, the political atmosphere after the 1997 Kyoto protocol was signed created strong incentives for countries to underreport industrial CO_2 emissions as well as agricultural ones. Such potential underreporting was not taken into account for this paper, so the actual emissions and sinks after 1997 are likely underestimated.

For the pre-1958 CO_2 concentrations, the author started with a CO_2 concentration 292 ppm for 1872-1882 from (Keeling & From, *Reassessment of late 19th century atmospheric carbon dioxide variations ...*, 1986), since this source seems the most reliable and methodologically consistent with the standard post-1958 data (the Keeling curve). (Keeling & From, 1986) declared a very high error margin of 10 ppm - equal to the total anthropogenic emissions for 1860-1888. From this point the author approximated the CO_2 concentrations for 1860-1957, assuming exponential increase with values of 292 ppm in 1876 and 315 ppm in 1958. This approach was selected because the only purpose of this work was to test the results of H2015, and a single datapoint around 1876 is sufficient for that task. Further, the extrapolated concentrations were compared with the ice core data from (Keeling & From, 1986), and found to match well. Nevertheless, the pre-1958 concentrations are not necessarily the best estimates, and are used only for the internal calculations and visualization in this paper, for which they are fully adequate. 1860 was selected as the start time because it is far enough from the 1815 Eruption of Mountain Tambora and because the anthropogenic CO_2 release was insignificant before that year.

3. Methods & Results

Validation of H2015 was performed in four ways:

1) Assuming a constant surplus CO_2 half-life over the whole period of 1860-2013, the equilibrium concentration and the half-life were best-fitted.
2) Since H2015 did not attempt a prediction for 150+ years, the pre-1958 half-life was allowed to vary linearly over 55 years, prior to 1958 (the period equal to 1958-2013, used in H2015 and H2015#1). The half-life was required to be constant pre-1903 and post-1958.
3) The same calculations were performed with the assumption that the historical LUCE was over-- or underestimated by 20%.
4) The results were compared to the best fit model for post-1958 only (H2015#1).

By definition, equilibrium is constant in each scenario, although its fitted estimates were different in different scenarios. In all four cases, the calculated equilibrium value fell within the range 275-285 ppm, perfectly matching proxy data for CO2 concentrations from 1000-1800 CE. The results for the different scenarios are shown in Table 1 in p.3.3.

3.1. Constant half-life for 1860-2013 and original LUCE data pre-1992

1860-2013 is longer than multi-decadal period of H2015. Nevertheless, the match between the predicted results and the data is excellent, as seen in Fig. 1 and Fig. 2.

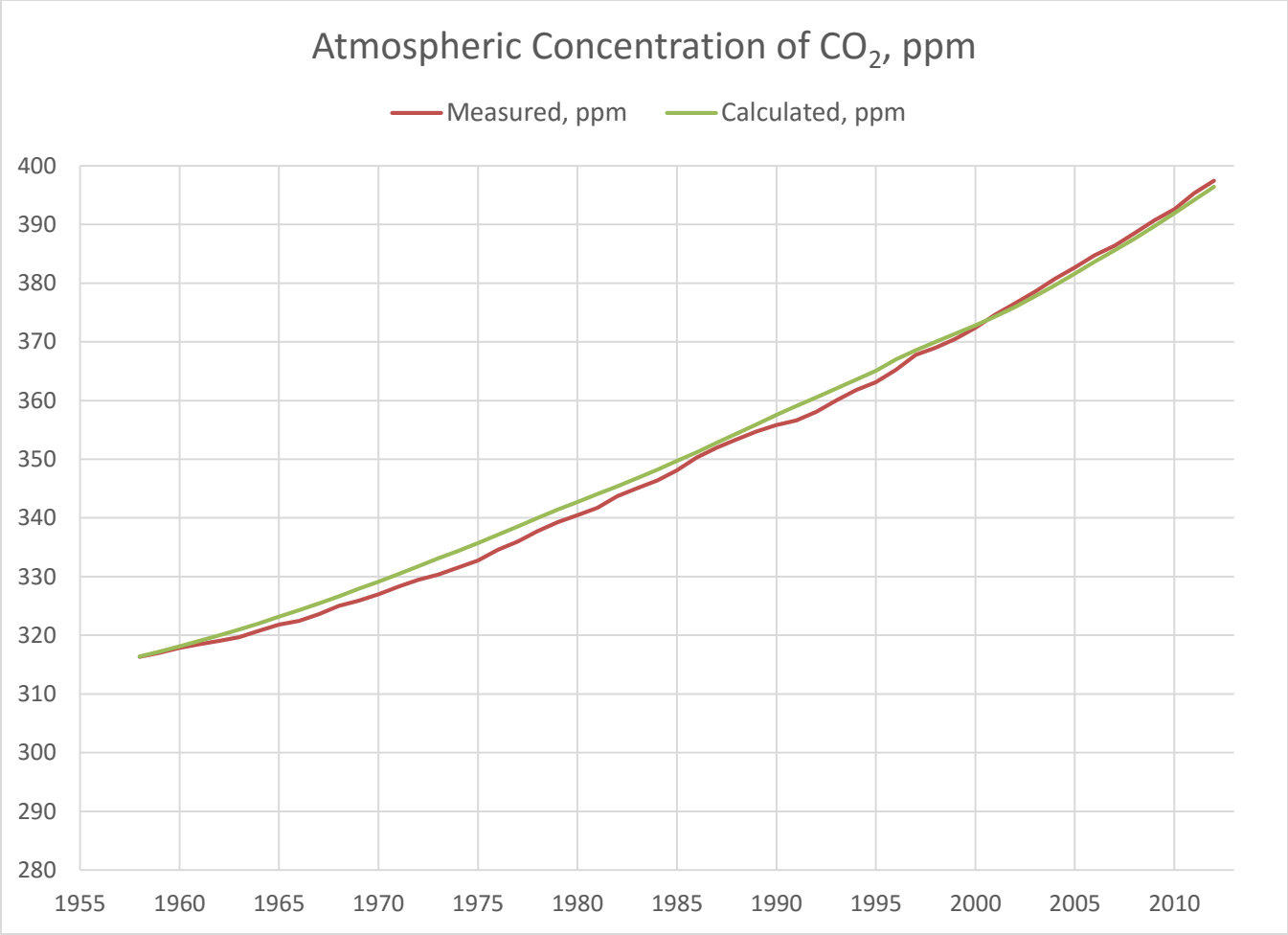

Fig. 1. Comparison of the measured concentration of CO_2 in the atmosphere (Keeling curve) to that computed using formula (2) with the same constant half-life from 1860-2013.

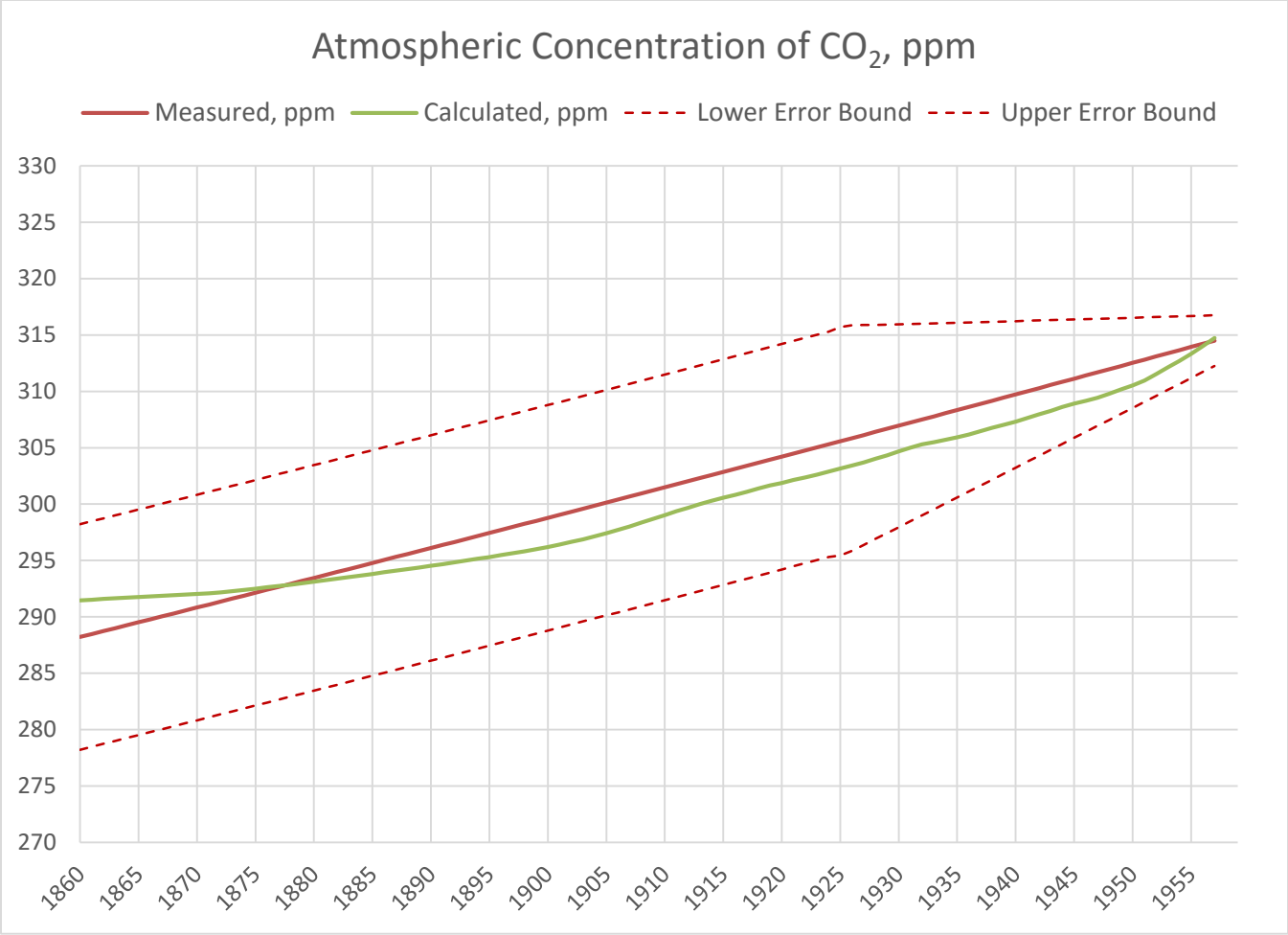

Fig. 2. Comparison of the concentrations of CO_2 in the atmosphere, based on historic measurements from 1872-1882 (Keeling & From, 1986) and computed using formula (2) with the same constant half-life, from 1860-1957.

3.2. Variable half-life and original LUCE data pre-1992

There is one constant half-life pre-1903, annual change at the rate of -0.4% per year from 1903-1958, and another constant half-life post-1958. The apparent rate of change from 1903-1958 is small and does not contradict the result of H2015. It may be explained by either physical and/or biological effects, excluded from consideration in H2015, or by inaccuracy in the data. The match is shown in Fig. 3 and Fig. 4.

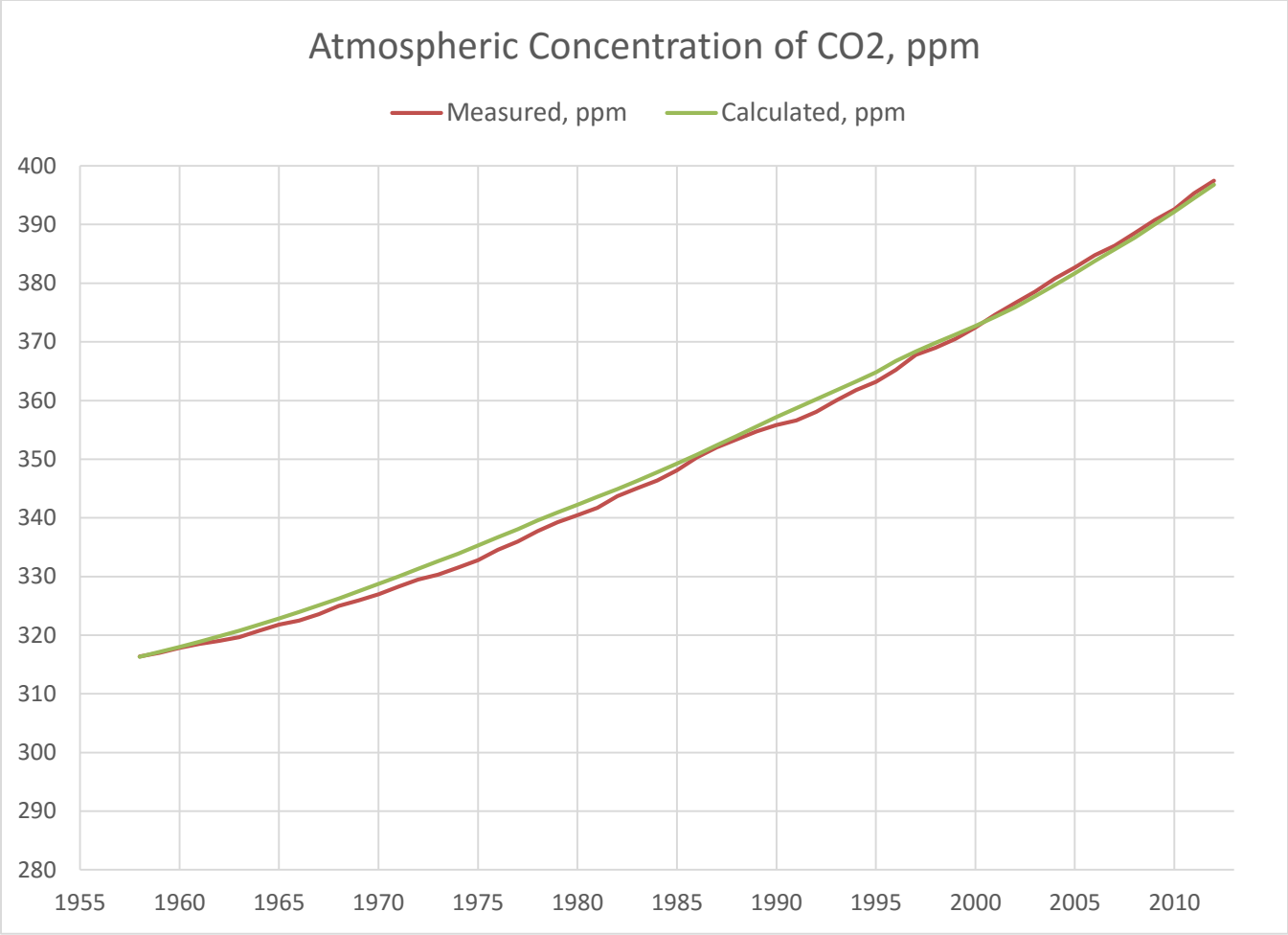

Fig. 3. Comparison of the measured concentration of CO_2 in the atmosphere (the Keeling curve) to that computed using formula (2) with a variable half-life from 1958-2013.

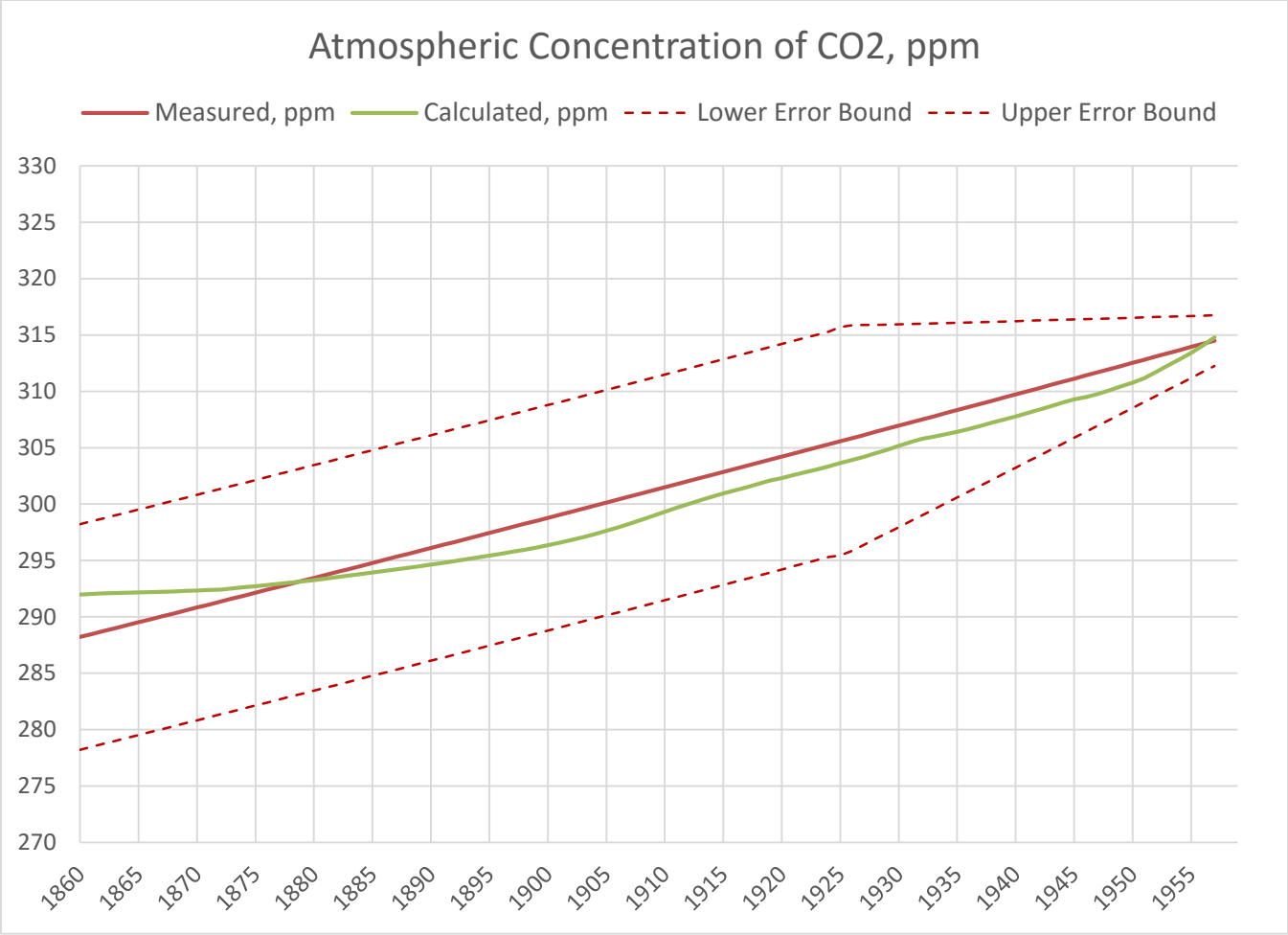

Fig. 4. Comparison of the concentrations of CO_2 in the atmosphere, based on historic measurements from 1872-1882 (Keeling & From, 1986) and data computed using formula (2) with a variable half-life, from 1860-1957.

3.3. LUCE error scenarios for 1860 - 2013

Table 1. Results for various scenarios of under- and overestimation of Land Use Change Emissions

Scenario	Approximation Type	Fitted half-time, years	Fitted Equilibrium, ppm	UMS of relative residuals, post 1958
LUCE is overestimated, correction=0.8	Constant Rate	**30**	285	5.0%
	Variable Rate	37 - **31**	283	5.0%
LUCE is OK	Constant Rate	**28**	283	4.9%
	Variable Rate	37 - **30**	280	4.8%
LUCE is overestimated, correction=1.25	Constant Rate	**27**	280	4.8%
	Variable Rate	37 - **29**	275	4.6%

UMS - unbiased mean squared

In all cases, the calculated pre-1958 concentrations are within the specified error margins of the estimated historical values. Unbiased mean squared values of the relative residuals (here, relative residuals are differences between the calculated and apparent sinks, divided by total emissions in one year) do not exceed 5%, clearly within the error margins of the data. These scenarios show that the results are not sensitive to the likely LUCE data errors. Of course, other LUCE error distributions are possible. For example, LUCE estimates might have a systematic bias, increasing further in the past. But since the LUCE fraction in the overall emissions has been steadily falling, the impact of such distributions would be similar to the analyzed ones.

Table 2. Values from H2015#1

Scenario	Approximation Type	Fitted half-time, years	Fitted Equilibrium, ppm
LUCE is OK	Constant post-1958	**35**	267

Finally, the results for the period 1860-2013 are close to those obtained in H2015#1 without pre-1958 data. If the half-life is allowed to vary, then the pre-1903 and post-1958 half-lives bracket the half-life from H2015#1.

4. Discussion

The graph in Fig. 5 shows CO_2 sink values, calculated according to formula (2), and compared with apparent sinks, calculated according to p.3.1 (constant sink rate over 1860-2013, original pre-1992 LUCE data), where the 5-year sink averages are weighted.

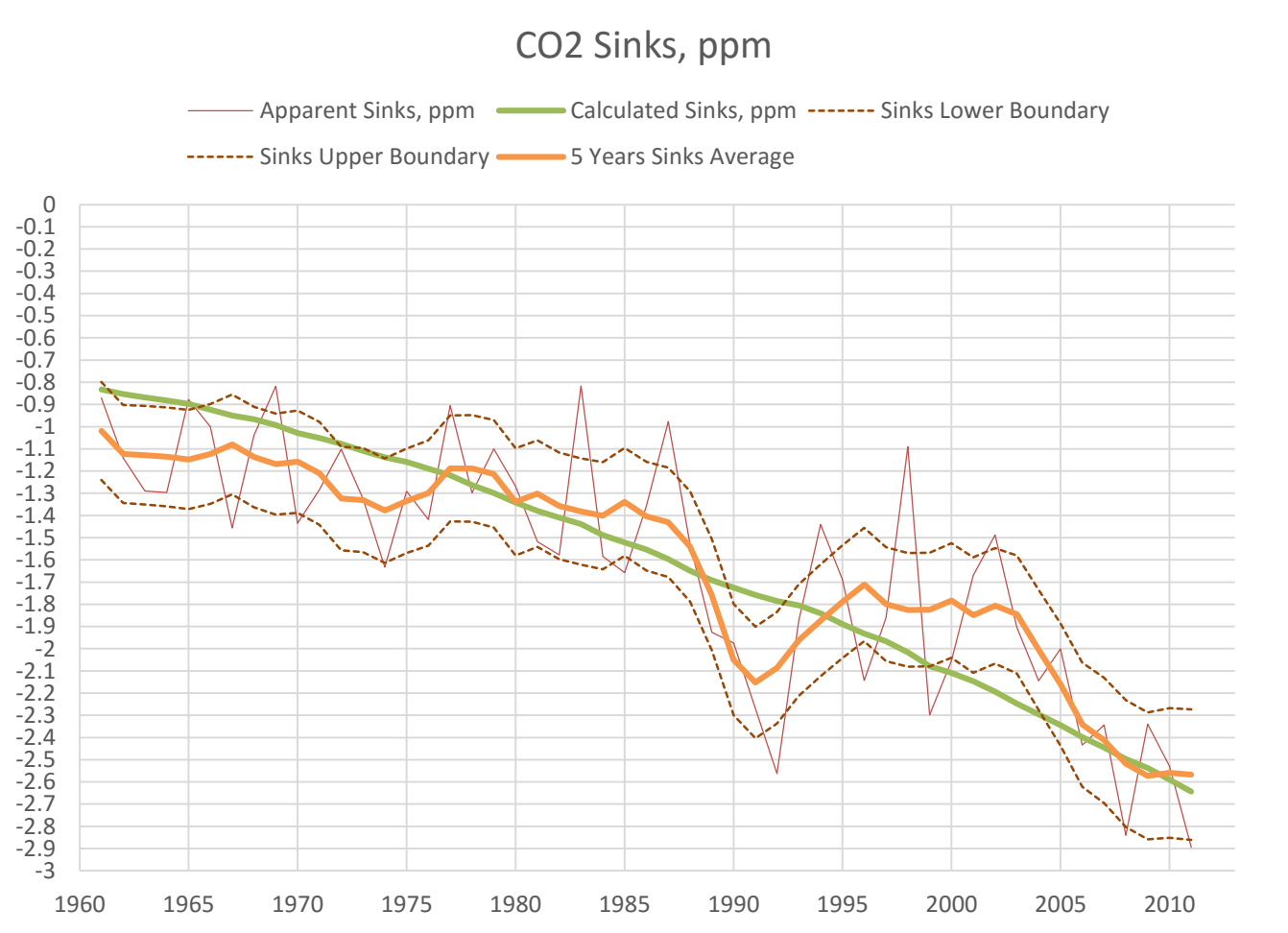

Fig. 5. Comparison of the apparent sinks from the paper dataset and the ones computed using formula (2), the constant half-life.

The error graph shows an excellent match between the theoretical results and the actual sinks. This is even more surprising, given the plentitude of the natural effects that might impact CO_2 sinks. In addition to the factors mentioned in H2015, current, future and recent changes in atmospheric CO_2 concentrations are likely be affected by climatic conditions that existed hundreds of years ago, including: atmospheric CO_2 concentrations, surface water temperatures, amounts of iron in the

epipelagic zone, and air-water mixing intensities (dependent on wind strength). The mechanism is the so-called "ocean memory" - ocean circulation brings to the surface the same water that sank hundreds years ago, and that water conserves some of its properties (probably including CO_2 concentration and heat content) to some degree. If significant, ocean memory should influence either or both air CO_2 concentrations and surface temperatures. The global impact of the ocean memory can vary slowly, over time periods of 20-200 years.

Among the two hypotheses for which calculations were made — a constant sink rate and an irregularly rising sink rate — the simpler hypothesis of the constant sink rate should be preferred.

5. Conclusions

This paper confirms the result of H2015: surplus CO_2 is removed from the atmosphere by natural sinks at a rate proportional to the surplus CO_2 concentration. In other words, a CO_2 pulse undergoes exponential decay with a single decay constant. It also provides a more accurate estimate of the half-life of the surplus CO_2 concentration, 30-35 years. The correspondent equilibrium concentration is estimated to be most likely in the range of 270-280 ppm (larger equilibrium concentrations correspond to lower half-lives). Possible errors in the land use change emissions data do not impact the results significantly.

6. Disclosure statement

The author declares an absence of conflicts of interest.

7. References

Boden, T. A., & Marland, G. (2013). Regional CO2 Emissions from Fossil-Fuel Burning, Cement Manufacture and Gas Flaring: 1899-2011. doi:10.3334/CDIAC/00001_V2010

Boden, T., Andres, B., & Marland, G. (2013). Global CO2 Emissions from Fossil-Fuel Burning, Cement Manufacture, and Gas Flaring: 1751-2010 [Live Dataset].

Halperin, A. (2015). Comment #1 on Simple Equation of Multi-Decadal Atmospheric Carbon Concentration Change. *defyccc.com*. http://defyccc.com/se/#1

Halperin, A. (2015). *Notes on FAO-FRA.* Retrieved from http://defyccc.com/notes-on-fao-fra-2010

Halperin, A. (2015). Simple Equation of Multi-Decadal Atmospheric Carbon Concentration Change. *defyccc.com*. http://defyccc.com/se

Halperin, A. (2016). *The IPCC has been Deceiving the Public about the Carbon Cycle from the Start.* Retrieved from Watts Up With That: http://wattsupwiththat.com/2016/03/16/the-ipcc-has-been-deceiving-the-public-about-the-carbon-cycle-from-the-start

Houghton, R. A. (2008). Carbon Flux to the Atmosphere from Land-Use Changes 1850-2005.

Keeling, C. D., & From, E. (1986). Reassessment of late 19th century atmospheric carbon dioxide variations in the air of western Europe and the British Isles based on an unpublished analysis of contemporary air masses by G. S. Callendar. *Tellus, 38B*, 87-105.

Keeling, C. D., Adams Jr., J. A., Ekdahl Jr., C. A., & Guenther, P. R. (1976). Atmospheric Carbon Dioxide Variations at the South Pole [Live Dataset]. *Tellus, 28*, 552-564.

Keeling, C. D., Piper, S. C., Bacastow, R. B., Wahlen, M., Whorf, T. P., Heimann, M., & A., M. H. (2001). Exchanges of atmospheric CO2 [Live Dataset]. *Scripps Institution of Oceanograph*.

Quéré, C. L. (2015). Global carbon budget 2014. *Earth Syst. Sci. Data, 7*(1), 47-85. doi:10.5194/essd-7-47-2015